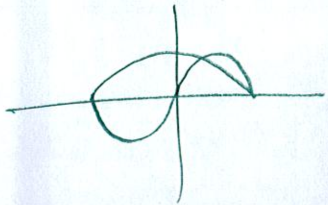
Find the area of the region between $y = \sin 2x$ and $y = \cos x$ for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$.

SCORE: _____ / 25 PTS

<u>Your final answer must be a number, not an integral. HINT: The answer is NOT 2.</u>



$\sin 2x = \cos x$

$2 \sin x \cos x = \cos x$

$2 \sin x \cos x - \cos x = 0$

$\cos x (2 \sin x - 1) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$x = \pm \frac{\pi}{2}$ $\quad$ $x = \frac{\pi}{6}$

$$\boxed{\int_{-\frac{\pi}{2}}^{\frac{\pi}{6}}} (\cos x - \sin 2x)\, dx + \boxed{\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}} (\sin 2x - \cos x)\, dx$$

$$= \left(\sin x + \tfrac{1}{2} \cos 2x\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{6}} + \left(-\tfrac{1}{2} \cos 2x - \sin x\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= \left(\tfrac{1}{2} + \tfrac{1}{4} - \left(-1 - \tfrac{1}{2}\right)\right) + \left(\tfrac{1}{2} - 1 - \left(-\tfrac{1}{4} - \tfrac{1}{2}\right)\right)$$

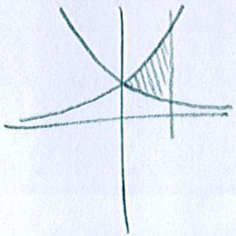
$$= \frac{9}{4} + \frac{1}{4}$$

$$= \frac{5}{2} \quad 4$$

3 EACH UNLESS OTHERWISE NOTED

Find the **y-coordinate** of the center of mass of the region defined by $y \le e^x$, $y \ge e^{-x}$ and $x \le 1$. **SCORE: _____ / 25 PTS**

Your final answer must be a number, not an integral.



$$\int_0^1 (e^x - e^{-x})\,dx$$

$$= (e^x + e^{-x})\Big|_0^1$$

$$= e + \frac{1}{e} - (1+1)$$

$$= \frac{e^2 - 2e + 1}{e}$$

$$= \frac{(e-1)^2}{e}$$

$$\frac{1}{2}\int_0^1 (e^{2x} - e^{-2x})\,dx$$

$$= \frac{1}{2}\left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)\Big|_0^1$$

$$= \frac{1}{4}\left(e^2 + \frac{1}{e^2} - (1+1)\right)$$

$$= \frac{1}{4}\left(\frac{e^4 - 2e^2 + 1}{e^2}\right)$$

$$= \frac{1}{4}\frac{(e^2-1)^2}{e^2}$$

3½ EACH

UNLESS

OTHERWISE

NOTED

$$\bar{y} = \frac{1}{4}\frac{(e^2-1)^2}{e^2} \cdot \frac{e}{(e-1)^2}$$

$$= \frac{(e+1)^2}{4e}$$

4

For a certain popular class, the registration window is only open for 5 days (to prevent excessive enrollment). **SCORE: _____ / 30 PTS**
Suppose a student is randomly selected among those who enrolled during that time. Let X be the time (measured in days) after registration opened that the student enrolled for the class. Suppose that the probability density function for X is given by

$$f(x) = \begin{cases} \dfrac{1}{(1+kx)^2}, & x \in [0, 5] \\ 0, & x \notin [0, 5] \end{cases}$$

[a] Find the probability that a student enrolled for the class during the last 24 hours that registration was open.

$\int_0^5 \frac{1}{(1+kx)^2}\,dx = 1$ 4

3 $-\frac{1}{k}\frac{1}{1+kx}\Big|_0^5 = 1$

$-\frac{1}{k}\left(\frac{1}{1+5k} - 1\right) = 1$

$-\frac{1}{\cancel{k}} \cdot -\frac{5\cancel{k}}{1+5k} = 1$

$5 = 1+5k$

$k = \frac{4}{5}$, 3

4 $\int_4^5 \frac{1}{(1+\frac{4}{5}x)^2}\,dx = $ 2 $-\frac{5}{4}(1+\frac{4}{5}x)^{-1}\Big|_4^5$

$= -\frac{5}{4}\left(\frac{1}{5} - \frac{5}{21}\right)$

$= -\frac{\cancel{5}}{\cancel{4}} \cdot \frac{-\cancel{4}}{\cancel{105}_{21}}$

$= \frac{1}{21}$ 3

[b] Find the median time that registration was open before a student enrolled in the class.

$\int_0^m \frac{1}{(1+\frac{4}{5}x)^2}\,dx = \frac{1}{2}$ 4

$-\frac{5}{4}(1+\frac{4}{5}x)^{-1}\Big|_0^m = \frac{1}{2}$

$\frac{1}{1+\frac{4}{5}m} - 1 = -\frac{2}{5}$ 3

$\frac{1}{1+\frac{4}{5}m} = \frac{3}{5}$

$1 + \frac{4}{5}m = \frac{5}{3}$

$\frac{4}{5}m = \frac{2}{3}$

$m = \frac{5}{6}$ DAYS 1

3

Find the length of the curve $\begin{array}{l} x = 3 - 12e^t \\ y = 9t - 2e^{2t} \end{array}$ for $t \in [0, 2]$. **SCORE: _____ / 20 PTS**

<u>**Your final answer must be a number, not an integral.**</u>

$$\int_0^2 \sqrt{(-12e^t)^2 + (9 - 4e^t)^2}\, dt$$

$$= \int_0^2 \sqrt{144e^{2t} + 81 - 72e^t + 16e^{2t}}\, dt$$

③ EACH UNLESS OTHERWISE NOTED

$$= \int_0^2 \sqrt{81 + 72e^t + 16e^{2t}}\, dt$$

$$= \int_0^2 (9 + 4e^{2t})\, dt$$

$$= (9t + 2e^{2t})\Big|_0^2 = 18 + 2e^4 - 2 = 16 + 2e^4$$ ②

The region bounded by $y = x^2 + 1$, $y = 18x - 80$ and $y = 10$ is revolved around the line $y = 2$. **SCORE: _____ / 35 PTS**

[a] Write, **BUT DO NOT EVALUATE**, a **single** integral for the volume of the resulting solid.

$x^2 + 1 = 18x - 80$

$x^2 - 18x + 81 = 0$ 3

$(x-9)^2 = 0$

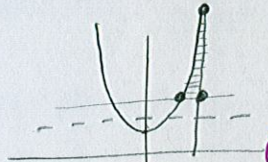
$x = 9 \rightarrow y = 82$

$x^2 + 1 = 10$

$x = \pm 3 \rightarrow y = 10$

$18x - 80 = 10$

$x = 5 \rightarrow y = 10$



$y = 18x - 80 \rightarrow x = \frac{y+80}{18}$

$y = x^2 + 1 \rightarrow x = \pm\sqrt{y-1}$

$$\underbrace{2\pi}_{2} \overbrace{\int_{10}^{82}}^{3} \underbrace{(y-2)}_{3} \underbrace{\left(\overset{3}{\frac{y+80}{18}} - \overset{3}{\sqrt{y-1}}\right)}_{3} dy$$

[b] If you used the disk or washer method in [a],
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the shell method.
If you used the shell method in [a],
write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the same volume using the disk or washer method.

$$\underbrace{\pi \overbrace{\int_3^5}^{2} \underbrace{\left(\overset{1\frac{1}{2}}{(x^2+1-2)^2} - \overset{1\frac{1}{2}}{(10-2)^2}\right)}_{1\frac{1}{2}} dx + \pi \underbrace{\int_5^9}_{2} \underbrace{\left(\overset{1\frac{1}{2}}{(x^2+1-2)^2} - \overset{1\frac{1}{2}}{(18x-80-2)^2}\right)}_{1\frac{1}{2}} dx}_{2}$$

A solid of revolution has volume $\int_2^4 \pi((\sqrt{y}+7)^2 - ((4-2y)+7)^2)\, dy$.

SCORE: _____ / 15 PTS

Find the equation of the axis of revolution, and the equations of the boundaries of the region being revolved.
Sketch & shade in the region being revolved.

Do NOT use the x– nor y–axes as boundaries nor the axis of revolution.

Equation of axis of revolution: (3) $x = -7$

Equations of boundaries:

(2) $x = \sqrt{y} \rightarrow y = x^2,\ x \geq 0$

(2) $x = 4 - 2y \rightarrow y = 2 - \frac{1}{2}x$

(1) $y = 2$

(1) $y = 4$